Chapter 21 : Coulomb's Law

- Electric charge
  ⇒ Two types of electric charge : Positive and negative charge.
- Electrically Neutral
  ⇒ The net charge is zero (positive charg = negative charge) balanced charge
- Electrically charged
  ⇒ Excess charge (unbalanced charges)

* Particles with the same sign of electrical charge repel each other, and particles with opposite signs attract each other.

- Coulomb's Law $\vec{F} = K \frac{q_1 q_2}{r^2} \hat{r} \Rightarrow \vec{F} = K \frac{|q_1||q_2|}{r^2}$
  ⇒ K = electrostatic constant "coulomb costant"
  $K = \frac{1}{4\pi \epsilon_0}$, ϵ_0 = permittivity constant
  $\epsilon_0 = 8.85 \times 10^{-12}$ $C^2/N.m^2$
  $K = 8.99 \times 10^{9}$ $N.m^2/C^2$

* $\vec{F}_{1,net} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \cdots + \vec{F}_{1n}$

- Electric charge is quantized (restriced to a certain values)
  $q = ne$, $e = 1.6 \times 10^{-19}$ C , $n = \mp 1, \mp 2, \mp 3, \cdots$

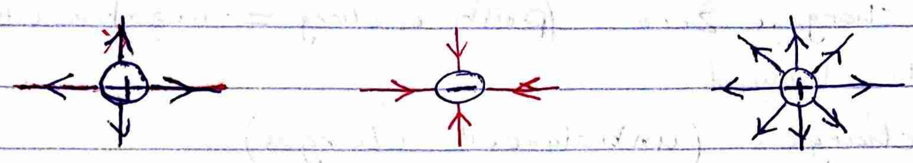
- Charge is conserved
  The net electric charge of any isolated system is always conserved.
  $e + e^{+} \rightarrow \gamma$ (لول)

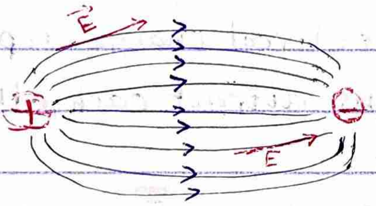
✓ Sample problem 21.04

Chapter 22 : Electric Fields

- $\vec{E} = \frac{\vec{F}}{q_0}$, q_0 : test charge $[\vec{E}] = N/C$

- Electric field lines :





اتجاه $\vec{E}$ مماس لخطوط المجال

1 Electric Field due to a charge particle

$$\vec{F_e} = K \frac{q\, q_0}{r^2} \hat{r}$$

$$\vec{E} = \frac{F_e}{q_0} = \frac{K\, q_0\, \hat{r}}{r^2}$$

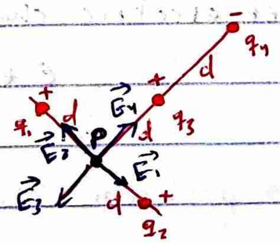
$$E = \frac{K\,|q|}{r^2}$$

2 Electric Field due to a set of charged particles

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \dots$$

Problem

(32) $q_1 = q_2 = +5e$, $q_3 = +3e$ and $q_4 = -12e$ / $d = 8.0\ \mu m$



$\rightarrow E_1 = E_2 \Rightarrow E_1 + E_2 = \text{zero}$

$\rightarrow E_3 = \frac{K q_3}{d^2} = \frac{9\times10^9 \times 3(1.6\times10^{-19})}{(8\times10^{-6})^2}$

$E_3 = 67.5\ N/C$

$\rightarrow E_4 = \frac{K q_4}{(2d)^2} = \frac{9\times10^9\,(12)\,(1.6\times10^{-19})}{(16\times10^{-6})^2}$

$E_4 = 67.5\ N/C$

$\Rightarrow E_3 + E_4 = \text{zero}$

$\therefore \boxed{\vec{E}_p = \text{zero}}$

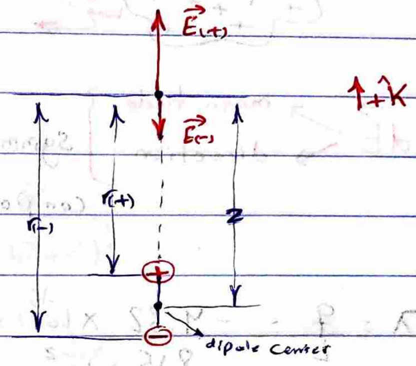
- The electric field due to a dipole :

⇒ An electric dipole cosists of two particles with charges of equal magnitude q but opposite signs, seperated by a small distance d :

- $\vec{p}$ = Electric dipble moment

$p = qd$ "from negative to positive"

⇒ $\vec{E}$ at point on a dipole axis



$E_+ = \dfrac{kq}{r_+^2}$ "upward"

$E_- = \dfrac{kq}{r_-^2}$ "downward"

$r_+ = z - \dfrac{d}{2} = z - a \quad , \quad a = \dfrac{d}{2}$

$r_- = z + \dfrac{d}{2} = z + a$

$$\Rightarrow E = E_+ - E_- = \frac{kq}{(z-a)^2} - \frac{kq}{(z+a)^2} = kq\left[\frac{1}{(z-a)^2} - \frac{1}{(z+a)^2}\right]$$

$$= kq\left[\frac{(z+a)^2 - (z-a)^2}{(z-a)^2(z+a)^2}\right] = kq\left[\frac{4za}{(z-a)^4}\right]$$

for $z \gg d$ ∴ $z \gg a$

$$\therefore E = kq\,\frac{4za}{z^4} = \frac{4kqa}{z^3}$$

$$a = \frac{d}{2} \rightarrow E = \frac{4kqd}{2z^3} = \frac{2kp}{z^3}$$

$$\therefore E = \frac{1}{2\pi\epsilon_0}\,\frac{p}{z^3} \quad \text{"upward"}$$

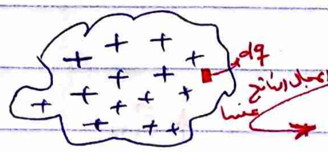
لأنه r_+ أقل من r_- وبالتالي $\vec{E}_{(-)} < \vec{E}_{(+)}$

لو عكسنا أماكن الشحنات ← نفس الجواب لكن سالب.

* نستخدم هذا القانون في حالة طلب المجال على امتداد الـ axis Dipole

3 $\vec{E}$ due to a continuous distribution $\Rightarrow$ integration

* $\lambda \equiv$ linear charge density C/m
  $\sigma \equiv$ Surface charge density C/m^2
  $\rho \equiv$ Volume charge density C/m^3



$dq \equiv$ charge element

$$dE = \frac{K\,dq}{r^2} \Rightarrow E = \int dE$$

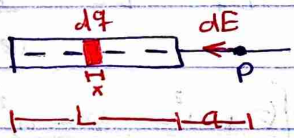
* $d\vec{E}$ → magnitude, direction } symmetry allows us to cancel out any of the components of the fields, to simplify the integration.

(13) (a) $\lambda = \frac{q}{L} = \frac{-4.32 \times 10^{-15}}{8.15 \times 10^{-2}} = -5.19 \times 10^{-14}$ C/m

(b) $dE = \frac{K\,dq}{x^2}$; $\Rightarrow \lambda = \frac{q}{x}$

$dE = K \frac{\lambda\,dx}{x^2}$; $q = \lambda x$ (constant λ)

$dq = \lambda\,dx$



"The charge extends from $x_1 = a$ to $x_2 = L + a$"

$$\Rightarrow E = \int_a^{L+a} K\lambda \frac{dx}{x^2} = K\lambda \int_a^{L+a} x^{-2}\,dx = K\lambda \left[\frac{-1}{x}\right]_a^{L+a}$$

$$E = K\lambda\left[\frac{-1}{L+a} + \frac{1}{a}\right] = K\lambda\left[\frac{\cancel{-a} + L + \cancel{a}}{a(L+a)}\right] = \frac{K\overbrace{\lambda L}^{q}}{a(L+a)}$$

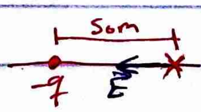
$$\therefore \boxed{\vec{E}_P = -\frac{Kq}{a(L+a)}\,\hat{i}} \xrightarrow{\text{بالتعويض}} \vec{E}_P = -4.48 \times 10^{-3}\ N/C\ \hat{i}$$

(d) $a = 50$ m

$$\vec{E}_P = -\frac{9\times10^{9}\,(4.23\times10^{-15})}{6\times10^{-2}(6\times10^{-2} + 8.15\times10^{-2})}\,\hat{i} = -1.52 \times 10^{-8}\ N/C\ \hat{i}$$

(e) charged particle

$$\vec{E} = \frac{Kq}{r^2} = \frac{9\times10^{9} \times 4.23\times10^{-15}}{(50)^2}$$



$$= -1.52 \times 10^{-8}\ N/C\ \hat{i}$$

* $a \gg L \Rightarrow a(L+a) = a^2$
  the rod treats as particle $\Rightarrow E = \frac{Kq}{a^2}$

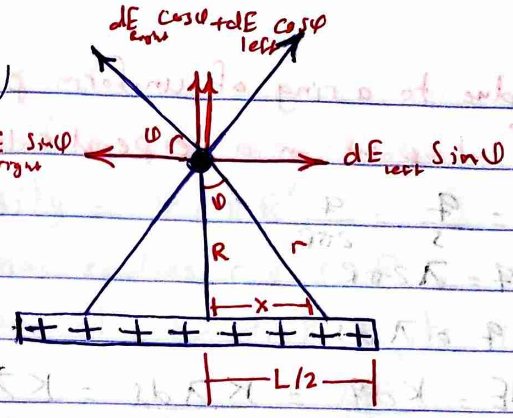
(10)

$dE_{left} = dE_{Right}$ (have same magnitude)

$dE_x = -dE_{Right}\sin\theta\,\hat{i} + dE_{left}\sin\theta\,\hat{i}$

$= \text{Zero}$ (by symmetric)

$\therefore \vec{E} = \int dE_y$



using $\lambda X = q$

$dq = \lambda\, dx$

$r^2 = R^2 + x^2$ and $\cos\theta = \frac{R}{r} = \frac{R}{(R^2+x^2)^{\frac{1}{2}}}$

$$\Rightarrow \vec{E} = \int \frac{K\,dq}{r^2}\cos\theta\,\hat{j} = \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{K\lambda\,dx}{(R^2+x^2)}\,\frac{R}{(R^2+x^2)^{\frac{1}{2}}}$$

$$= 2K\lambda R\,\hat{j}\int_0^{L/2} \frac{dx}{(R^2+x^2)^{3/2}}$$

تكامل بالتعويض

في الامتحانات يجيبو تكاملات بسيطة بدون أجزاء أو فرعها

$$= 2K\lambda R\,\hat{j}\int \frac{R\sec^2\theta\,d\theta}{R^3\sec^3\theta}$$

$\rightarrow \tan\theta = \frac{x}{R} \rightarrow x = R\tan\theta$

$\boxed{dx = R\sec^2\theta\,d\theta}$

$$= \frac{2K\lambda}{R}\,\hat{j}\int \frac{d\theta}{\sec\theta}$$

$\rightarrow (R^2+x^2)^{3/2} = (R^2+R^2\tan^2\theta)^{3/2}$

$= (R^2\sec^2\theta)^{3/2}$

$= \boxed{R^3\sec^3\theta}$

$$= \frac{2K\lambda}{R}\,\hat{j}\,\sin\theta$$

use $\sin\theta = \frac{x}{(R^2+x^2)^{1/2}}$

$$\Rightarrow \vec{E} = \left[2K\lambda\,\frac{x}{(R^2+x^2)^{1/2}}\,\hat{j}\right]_0^{L/2} = \frac{K}{R}\,\frac{\lambda L}{(R^2+(L/2)^2)^{1/2}}\,\hat{j}$$

Substituted $q = 9.25\times10^{-12}$, $L = 16\text{ cm} = 0.16\text{m}$ / $R = 0.06\text{m}$

$$\boxed{\vec{E} = 13.86\ \text{N/C}\ \hat{j}}$$

- $\vec{E}$ due to a ring of uniform positive charge

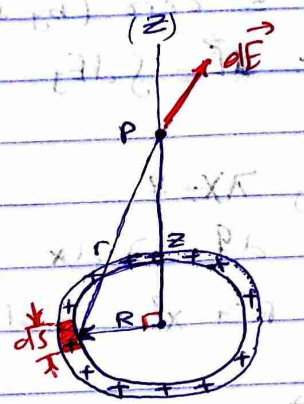
$\Rightarrow \vec{E}$ at point on a perpendicular axis to the ring plane

$\lambda = \frac{q}{s} = \frac{q}{2\pi R}$

$q = \lambda 2\pi R$

$dq = \lambda ds$

$\Rightarrow dE = \frac{K dq}{r^2} = \frac{K \lambda ds}{r^2} = \frac{K \lambda ds}{R^2 + z^2}$



By symmetry the component perpendicular to the z-axis cancel.

$$E_{ring} = \int dE \cos\theta = \int \frac{K \lambda ds}{R^2 + z^2} \frac{z}{(z^2+R^2)^{1/2}}$$

$$= \frac{K z \lambda}{(z^2+R^2)^{3/2}} \int_0^{2\pi R} ds$$

$$= \frac{K z \lambda \, 2\pi R}{(z^2+R^2)^{3/2}}$$

total charge of the ringe $\Rightarrow q = \lambda(2\pi R)$

$$= \frac{K q z}{(z^2+R^2)^{3/2}}$$

$$\boxed{\vec{E}_{ring} = + \frac{K q z}{(z^2+R^2)^{3/2}} \hat{k}}$$

$\Rightarrow z \gg R \Rightarrow E = \frac{Kq}{z^2}$ (like charged point particle)

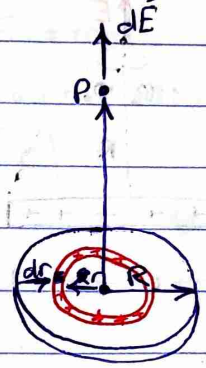
* E at the center of ring = Zero

- $\vec{E}$ due to a charged disk

$\Rightarrow$ A disk of radius R and uniform positive charge.

$\sigma \equiv$ surface charge density $= q/\pi R^2$

- Take a ring on the disk has radius r; $r \leq R$, then integrate from the center of the disk to its rim $\Rightarrow (0 \to R)$



$$\Rightarrow dE = \frac{dq\, z}{4\pi\epsilon_0 (z^2+r^2)^{3/2}}$$ (in the positive direction of the z-axis)

use $\sigma = \frac{q}{A}$

~~$A = q/\sigma$~~

$q = \sigma A$

$dq = \sigma\, dA = \sigma(2\pi r\, dr)$

$$\Rightarrow E = \int dE = \frac{\sigma z}{4\epsilon_0} \int_0^R 2r\, dr\, (z^2+r^2)^{-3/2}$$

use $\int x^m\, dx = \frac{x^{m+1}}{m+1}$ $\quad x = z^2+r^2$ / $m = -3/2$ / $dx = 2r\,dr$

$$E = \frac{\sigma z}{4\epsilon_0}\left[\frac{(z^2+r^2)^{-1/2}}{-\frac{1}{2}}\right]_0^R$$

$$= \frac{\sigma z}{2\epsilon_0}\left[\frac{(z^2+R^2)^{-\frac{1}{2}} - z^{-1}}{-\frac{1}{2}}\right]$$

$$\boxed{E_{disk} = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{z}{\sqrt{z^2+R^2}}\right]}$$

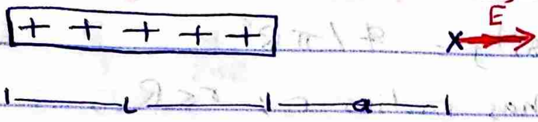
* $R \to \infty$

$$E_{disk} = \frac{\sigma}{2\epsilon_0}$$

✓ Sample problem 22.03

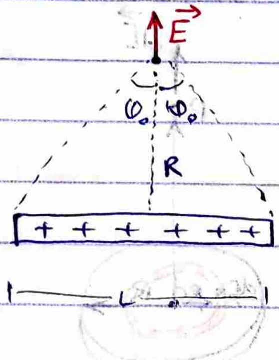
* Conclusion

①



$$\vec{E} = \frac{q}{4\pi\epsilon_0\, a(L+a)}$$

②

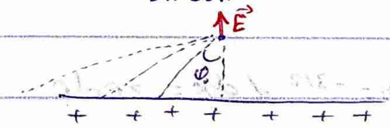


$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 R} \sin\phi_0$$

③ $\vec{E}$ due to an infinite line of charge

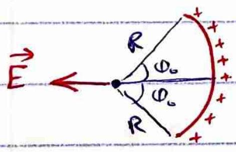
$L \to \infty \quad \therefore \quad \phi_0 \to \frac{\pi}{2}$

$$E = \frac{\lambda}{2\pi\epsilon_0 R} \sin\left(\frac{\pi}{2}\right)$$



$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 R}, \quad L \ggg R$$

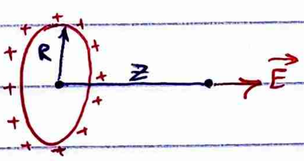
④ Uniformly charged circular arc



$$\vec{E} = \frac{\lambda}{2\pi\epsilon_0 R\, 2}\, 2\sin\phi_0$$

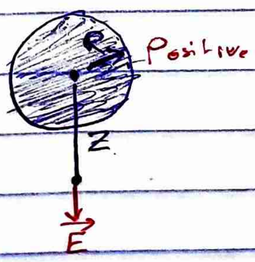
$$= \frac{2K\lambda}{R} \sin\phi_0$$

⑤ Uniformly charged Ring



$$\vec{E} = \frac{qZ}{4\pi\epsilon_0 (Z^2+R^2)^{3/2}}$$

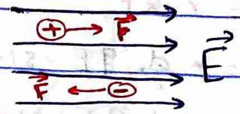
⑥ Uniformly charged Disk



$$\vec{E} = \frac{\sigma}{2\epsilon_0}\left[1 - \frac{Z}{\sqrt{Z^2+R^2}}\right]$$

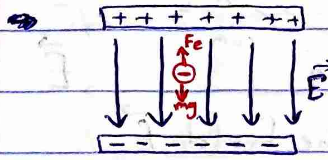
4 A Point charge in an Electric field

- $\vec{F} = q\vec{E}$, في هذا القانون نعوّض الإشارة



- Millikan's oil-drop experiment for measuring elementary charge (e)

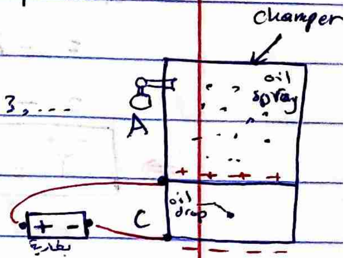
→ A negatively charged oil drop is suspended between the plates.



$\vec{F}_{oildrop} = \text{Zero} \Rightarrow qE = mg$

$q = \frac{mg}{E}$, $q = ne$, $n = 0 \mp 1 \mp 2 \mp 3, \ldots$

$\boxed{e = 1.6 \times 10^{-19}\ C}$



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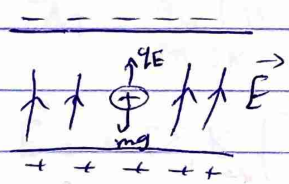


a+b $E = mg / q = 6.64\times10^{-27}(9.8) / 3.2\times10^{-19} = 2.03\times10^{-7}$ N/C upward

$\boxed{\vec{E} = 2.03\times10^{-7}\ N/C\ \hat{k}}$

c E is doubled, $E = 4.06\times10^{-7}$ N/C upward

$\vec{F}_{net} = q\vec{E} + m\vec{g} \Rightarrow ma = qE - mg$

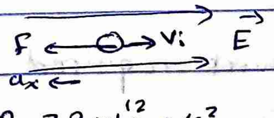
$a = \frac{qE - mg}{m} = \frac{(3.2\times10^{-19})(4.06\times10^{-7}) - (6.64\times10^{-27})(9.8)}{6.64\times10^{-27}}$

$\boxed{a = 9.77\ m/s^2}$ upward.

38 a $\vec{v}_f = \vec{v}_i + \vec{a}t$

$v_{ix} = 30$ km/s / $\vec{E} = +50$ N/C $\hat{i}$ (right ward)

$\vec{F} = q\vec{E} \Rightarrow m_e\vec{a} = -e\vec{E}$



$\therefore a_x = \frac{-eE}{m_e} = \frac{(-1.6\times10^{-19})(50\ N/C)}{9.11\times10^{-31}\ kg} = -8.78\times10^{12}\ m/s^2$

تنتبهوا تحوّلوه ل SI units

$\Rightarrow v_{fx} = v_{ix} + a_x t = (3\times10^{4}\ m/s) + (-8.78\times10^{12})(1.5\times10^{-9}\ s)$

$v_{fx} = 1.64\times10^{4}$ m/s , $\boxed{v_f = +1.64\times10^{4}\ m/s\ \hat{i}}$

b $\Delta x = v_{ix}t + \frac{1}{2}a_x t$

$\Delta x = (3\times10^{4})(1.5\times10^{-9}) + \frac{1}{2}(-8.78\times10^{12})(1.5\times10^{-9})^2$

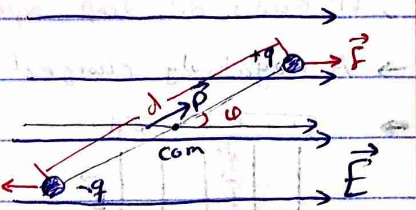
$\boxed{\Delta x = 3.51\times10^{-5}\ m}$

5 A dipole in An electric field

- $\vec{T} = \vec{r} \times \vec{F}$

$T_+ = -\frac{d}{2} qE \sin\theta$ "Clockwise"

$T_- = -\frac{d}{2} qE \sin\theta$ "Clockwise"

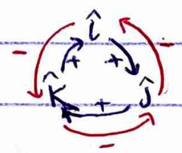


$T_{net} = \vec{T}_+ + \vec{T}_-$

$= -2 \frac{d}{2} qE \sin\theta$ (P)

$= -PE \sin\theta$

$\Rightarrow \boxed{\vec{T} = \vec{P} \times \vec{E}}$ Toarque on a dipole by the field $\vec{E}$

* 

$\vec{a} \times \vec{b} \neq -\vec{b} \times \vec{a}$

- A potential energy U is associated with the orientation of the dipole moment in the field $\boxed{U = -\vec{P}.\vec{E}}$ اتجاه

$\Rightarrow U = \begin{cases} PE \text{ (greatest value)}, & \vec{P} \text{ and } \vec{E} \text{ antiparallel } (\theta = 180°) \\ \text{zero} & \vec{P} \text{ and } \vec{E} \text{ perpendicular } (\theta = \pi/2) \\ -PE \text{ (least value)}, & \vec{P} \text{ and } \vec{E} \text{ parallel } (\theta = 0°) \end{cases}$

- If the diploe orientation changes $\Rightarrow$ * الشغل المبذول من حقل الكهرباء

The work done by the electric field $W_E = -\Delta U$

- If the change in the orientation is due to an external agent

$W_{external\ agent} = +\Delta U$ * الشغل اللازم

$W_{required} = +\Delta U$

41 work required $\rightarrow$ $W_{external} = +\Delta U$, $U = -PE\cos\theta$

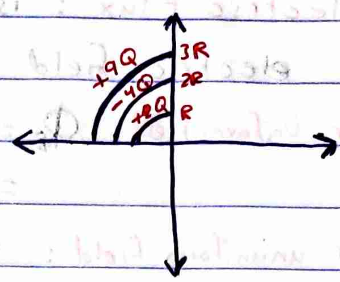
$W = -PE[\cos(23°+180°) - \cos(23°)]$

$= -3.02 \times 10^{-25}\ C.m \times 46.0\ N/C\ (-1.84)$

$= \boxed{2.56 \times 10^{-23}\ J}$

(59) To simplify a problem, rotate x-y $\frac{\pi}{4}$ clock wise.

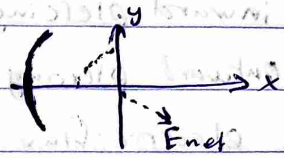
use $E = \frac{K\lambda}{r} \sin\theta \Big|_{-\frac{\pi}{4}}^{\pi/4} = \sqrt{2}\frac{K\lambda}{r}$



$\lambda = \frac{Q}{\text{length}}$, length $= \frac{2\pi r}{4}$

$\lambda_1 = \frac{Q(4)}{\frac{2\pi R}{4}} = \frac{2Q}{\pi R} \quad (+x)$

$\lambda_2 = \frac{4Q(4)}{2\pi 2R} = \frac{4Q}{\pi R} \quad (-x)$



$\lambda_3 = \frac{9Q(4)}{2\pi\, 3R} = \frac{6Q}{\pi R} \quad (+x)$

$\vec{E} = \sqrt{2}\,K\left[\frac{2Q}{\pi R^2} - \frac{4Q}{\pi R(2R)} + \frac{6Q}{\pi R(3R)}\right]$

$\vec{E} = 2\sqrt{2}\frac{KQ}{\pi R^2}$

$\vec{E} = 1.3\times10^{7}$ N/C (-45°)

(23) 10 g block / $q = +8\times10^{-5}$ C is placed in $\vec{E} = (3000\hat{i} - 6000\hat{j})$ N/C.

a) $\vec{F} = qE = 8\times10^{-5}(3000\hat{i} - 6000\hat{j})$

$= 0.24\hat{i} - 0.48\hat{j}$

$|\vec{F}| = \sqrt{(0.24)^2 + (-0.48)^2}$ $\quad \theta = \tan^{-1}\left(\frac{-0.48}{0.24}\right) = -63.4^\circ$

b) block is released أفلتت $v_i = 0$

$x_f - \cancel{x_i} = \cancel{v_{ix} t} + \frac{1}{2} a_x t^2$

$x = \frac{1}{2} a_x t^2$ $\quad a_x = \frac{F_x}{m}$

$= \frac{0.24\,(3)^2}{2\,(0.01)} = \boxed{108\text{ m}}$

* المفروض احسب a_x , a_y لحال
وبعدين بجيب السرعة

$F_y = -0.48$, $y = \frac{-0.48\,(3)^2}{2(0.01)} = \boxed{-216\text{ m}}$

c) $v_F = v_i + at$

$v_{Fx} = \cancel{v_{ix}} + a_x t$, $v_{Fy} = \cancel{v_{iy}} + a_y t$

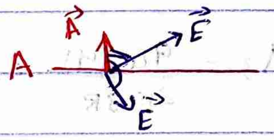
$S = \sqrt{(v_{Fx})^2 + (v_{iy})^2} = 161$ m/s

Chapter 23 : Gauss' Law

*هنا الشابتر كل الرسمات فيه 3D

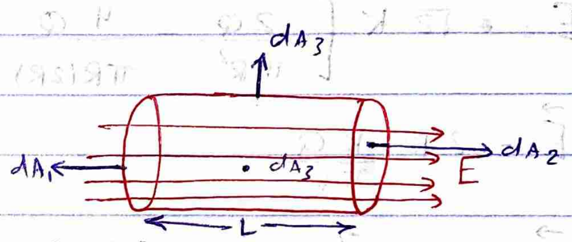
- Electric Flux (التدفق) : is directly proportional to the number of electric field lines piercing (تخترق) the surface Perpendicularly
- for Uniform field : $\Phi_E = \vec{E}\cdot\vec{A}$ $N.m^2/C$
  $= EA\cos\theta$ $N.m^2/C$
- for ununiform field : $\Phi_E = \int \vec{E}\cdot d\vec{A}$

* An inward piercing field is negative flux ($180° > \theta > 90°$)
An outward piercing field is positive flux ($90° > \theta > 0$)
The electric flux $= 0$ fo $\theta = 90°$
موازي للسطح
Skimming field has zero flux.



✓ Checkpoint 1

✓ Sample Problem 23.01

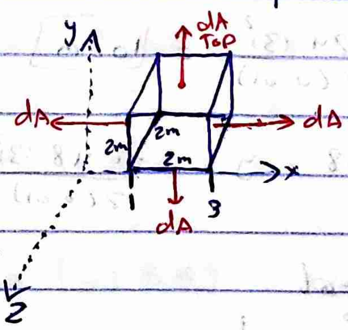


$(\Phi_E)_1 = \vec{E}\cdot\vec{A} = E\pi R^2\cos 180° = -E\pi R^2$
$(\Phi_E)_2 = \vec{E}\cdot\vec{A} = E\pi R^2\cos 0° = E\pi R^2$
$(\Phi_E)_3 = \vec{E}\cdot\vec{A} = E(2\pi RL)\cos 90° = 0$

$(\Phi_E)_{cylinder} = \Phi_{E1} + \Phi_{E2} + \Phi_{E3} = 0$, cylinder is closed surface
التدفق على الاسطوانة صفر يعني عدد خطوط المجال اللي دخلت يساوي اللي طلعت.

✓ Sample Problem 23.02 (Cube)
$\vec{E} = (3x)\hat{i} + 4\hat{j}$ N/C $\rightarrow E_x = 3x$ No uniform component
$\rightarrow E_y = 4$ uniform component

1) right face $d\vec{A} = \hat{i}\,dA$
$\Phi_r = \int \vec{E}\cdot d\vec{A} = \int(3x\hat{i} + 4\hat{j})\cdot\hat{i}\,dA$
$= \int(3x\,dA)(\hat{i}\cdot\hat{i})^{1} + \cancel{\int 4\,dA\,\hat{j}\cdot\hat{i}}^{\,0}$
$= \int 3x\,dA$
$= 9\int dA = 9(2)^2 = 36$ $N.m^2/C$



2) left face $d\vec{A} = -\hat{i}\,dA$
$\Phi_L = \int \vec{E}\cdot d\vec{A}$
$= -\int 3x\,dA = -3\int dA = -3(2)^2 = -12$ $N.m^2/C$

3) top face $\vec{dA} = \hat{j}\, dA$

$\Phi_{Top} = \int \vec{E} \cdot \vec{dA}$

$= \int (3x\hat{i} + 4\hat{j}) \cdot \hat{j}\, dA$

$= \int (3x\, dA)\cancel{(\hat{i} \cdot \hat{j})}^{0} + \int 4 dA \hat{j} \cdot \hat{j}$

$= \int 4\, dA$

$= 4\,(2)^2 = 16\ N.m^2/C$

Addition: 4) bottom face

$\Phi_{bottom} = -16\ N.m^2/C$

5) front face

$\Phi_f = \int (3x\hat{i} \cdot 4\hat{j}) \cdot \hat{k}\, dA = 0$

6) back face

$\Phi_b = \int (3x\hat{i} \cdot 4\hat{j}) \cdot -\hat{k}\, dA = 0$

$\therefore \Phi_{cube} = \Phi_r + \Phi_l + \Phi_t + \Phi_b + \Phi_f + \Phi_{back}$

$= 36 - 12 + 16 - 16 + 0 + 0$

$= \boxed{24\ N.m^2/C}$

- **Gauss' Law:**

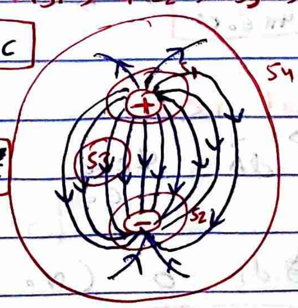
The electric flux through a closed surface equal to the total charge inside the surface divided by ϵ_0!

مهم نعوض بالإشارة * $\boxed{\Phi_{E\ \text{closed surface}} = \dfrac{q_{enc}}{\epsilon_0}}$ or $\boxed{\oint \vec{E} \cdot dA = \dfrac{q_{enc}}{\epsilon_0}}$

الشحنة الموجودة داخل السطح

- **Example:**

$q_1 = 17.7\mu C$, $q_2 = -17.7\ \mu C$, find Φ_{S1}, Φ_{S2}, Φ_{S3}, Φ_{S4}.



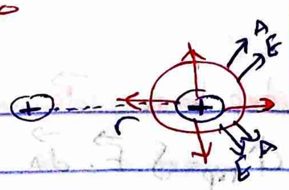
$\Phi_{S1} = \dfrac{q_{enc}}{\epsilon_0} = \dfrac{+17.7 \times 10^{-6}}{8.85 \times 10^{-12}} = \boxed{2 \times 10^{6}\ N.m^2/C}$

$\Phi_{S2} = \dfrac{q_{enc}}{\epsilon_0} = \dfrac{-17.7 \times 10^{-6}}{8.85 \times 10^{-12}} = \boxed{-2 \times 10^{6}\ Nm^2/C}$

$\Phi_{S3} = \dfrac{q_{enc}}{\epsilon_0} = \boxed{0}$ (صفر) لا شحنة عليها

$\Phi_{S4} = \dfrac{q_{enc}}{\epsilon_0} = \dfrac{(+17.7 - 17.7) \times 10^{-6}}{8.85 \times 10^{-12}} = \boxed{0}$

$$\Phi = \vec{E}\cdot\vec{A}$$
$$\Phi = \int \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$



* $E = \frac{kq}{r^2}$

$$\Rightarrow \Phi = \vec{E}\cdot\vec{A} = \frac{q}{\epsilon_0}$$

$$E\cdot(4\pi r^2) = \frac{q}{\epsilon_0}$$

$$\boxed{E = \frac{kq}{r^2}} \checkmark$$

- Applications on Gauss' Law

I Volume symmetry

قشرة كروية

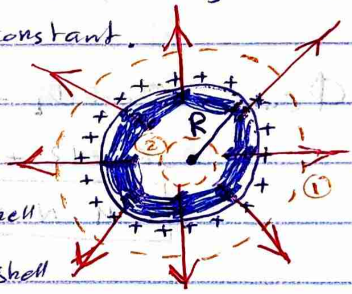
① E due to a Uniformly charged spherical shell

A spherical shell

radius $= R$ / charge $= Q$ / $A = 4\pi R^2\ m^2$ / $V = \frac{4}{3}\pi R^3\ m^3$

density $= \frac{Q}{4\pi R^2}$ / σ surface charge = constant.

* The charge distributed on the surface of the sphere



Exp ⇒ Find E at ① $r > R$, outside the spherical shell

② $r < R$, inside the sphercal shell

③ $r = R$.

(20)

① E at $r > R$

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$
$$\oint E\cos\theta\, dA = \frac{Q}{\epsilon_0}$$
$$E\oint dA = \frac{Q}{\epsilon_0}$$
$$E\, 4\pi r^2 = \frac{Q}{\epsilon_0}$$

$\boxed{E = \frac{Q}{4\pi\epsilon_0 r^2}}$, $r > R$ ∴ ③ $\boxed{E = \frac{Q}{4\pi\epsilon_0 R^2}}$ $r = R$

② E at $r < R$

$$\oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$ صافي الشحنة صفر

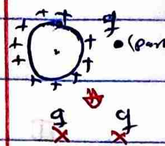
$E \oint dA = 0$ (No charge inside the sphere

$\boxed{E = 0 \text{ at } r < R}$

- The two shell theories for electrostatics

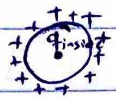
1. Shell theorem 1 :

الجسم المشحون خارج قشرة موزع عليها الشحنة بانتظام ، ينجذب أو يتنافر مع شحنة القشرة ، وكأنه نستطيع أن نعوضها بجسيم مشحون مكان القشرة المشحونة.



2. Shell theorem 2 :

الجسم المشحون داخل قشرة موزع عليها الشحنة بانتظام ، لا تؤثر عليها أي مجموعة قوى.

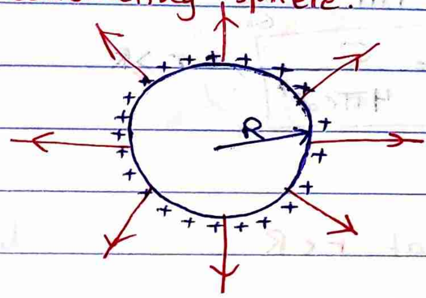


$E_{inside} = Zero$

$F = q\cancel{E}^{0} = 0$

② E due to a Uniformly charged Solid conducting sphere.

A conducting sphere of radius = R
is charge by a charge +Q
① find E at $r > R$
② find E at $r = R$
② find E at $r < R$



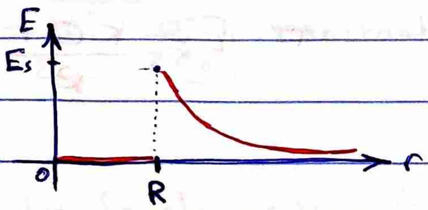
* Charged conducting sphere will behave as a spherical shell (تتصرف)

$\sigma = \frac{Q}{4\pi R^2}$ (charged conducting sphere)

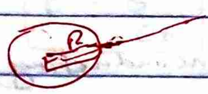
$E = \frac{Q}{4\pi\epsilon_0 r^2}$, $r > R$

$E_{surface} = \frac{Q}{4\pi\epsilon R^2}$, $r = R$

$E = 0$ inside the charged conductor



(E due to a charged conducting sphere)

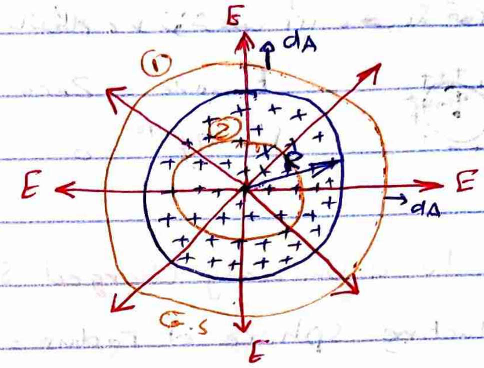


③ E due to a Uniformly charged Nonconducting sphere

radius = R / charge = Q / The volum charge density $\rho = \frac{Q}{\text{Volume}} = \frac{Q}{\frac{4}{3}\pi R^3}$

The charges distributed uniformly through its volume frome the center to the outer surface.

1) E at r>R



$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$

$\oint E \cos 0 \, dA = \frac{Q}{\epsilon_0}$

$E \oint dA = \frac{Q}{\epsilon_0}$

$E \, 4\pi r^2 = \frac{Q}{\epsilon_0}$

$\boxed{E = \frac{Q}{4\pi\epsilon_0 r^2}}$, r>R

2) E at r<R

$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$ ⟶ $q_{enc} = \int \overset{\text{constant}}{\rho} \, dv$

$= \rho \int dv$

$= \rho V$

$E \cancel{4\pi} r^{\cancel{2}} = \frac{\rho V}{\epsilon_0} = \frac{\rho \frac{4}{3}\cancel{\pi} r^{\cancel{3}}}{\epsilon_0}$

$\boxed{E = \frac{\rho r}{3\epsilon_0}}$, r<R, $\rho = \frac{Q}{\frac{4}{3}\pi R^3}$

$E = \left[\frac{Q}{\frac{4}{3}\pi R^3}\right] \cdot \frac{r}{3\epsilon_0}$

$\boxed{E = \frac{Qr}{4\pi\epsilon_0 R^3}}$ ②, r<R

* $E = \frac{kQ}{r^2}$, r>R

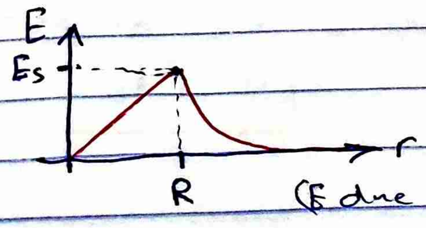
if r=R ⟶ $E = \frac{kQ}{R^2}$

if r<R ⟶ E=0

* $E = \frac{kQr}{R^3}$, r<R nonconducting

③ E at r=R from ① or ②

$\boxed{E_{surface} = \frac{Q}{4\pi\epsilon_0 R^2}}$



(E due to ~~...~~ Uniformy Nonconducting)

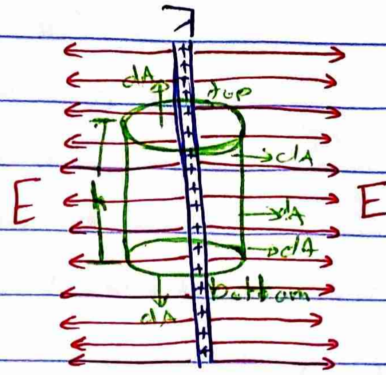
II Linear Symmetry

$\lambda = \dfrac{q}{h}$ / radius $= r$ / length $= h$

$\Rightarrow E_{top} = 0$, $\vec{E} \perp \vec{A}$
$E_{bottom} = 0$, $\vec{E} \perp \vec{A}$ $\Big\}$ $\cos 90 = 0$

$\therefore$ from curved surface

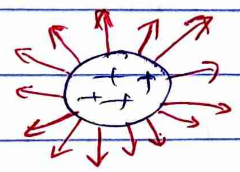
$A = 2\pi r h$



$\oint \vec{E} \cdot d\vec{A} = \dfrac{q_{enc}}{\epsilon_0}$ نعرف الشحنة داخل سطح جاوس فقط

$E \oint dA = \dfrac{\lambda h}{\epsilon_0}$, $\cos 0 = 1$

Top view



$E(2\pi r h) = \dfrac{\lambda h}{\epsilon_0}$

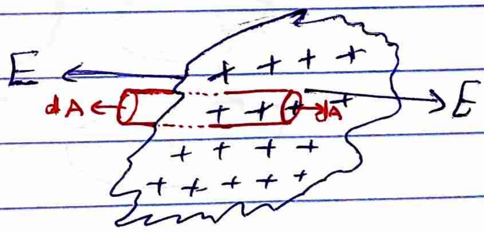
$\boxed{E = \dfrac{\lambda}{2\pi\epsilon_0 r}}$ very long charged rod.

III planar Symmetry

$\Rightarrow$ E due to a Uniformly charged, thin, infinite, Non conducting sheet (surface)

surface charge density $= \sigma$ C/m² in on surface

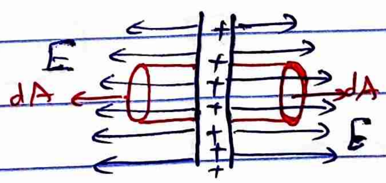
$\oint \vec{E} \cdot d\vec{A} = \dfrac{q_{enc}}{\epsilon_0}$, $\sigma = \dfrac{q}{A}$



$E\cos 0 \cancel{A} + E\cos 0 \cancel{A} = \dfrac{\sigma \cancel{A}}{\epsilon_0}$

$2E = \dfrac{\sigma}{\epsilon_0}$

rright view



$\boxed{E = \dfrac{\sigma}{2\epsilon_0}}$ constant

✓ Solve Sample problem (23.03)
(23.04)
(23.05)

$\Rightarrow$ conducting

$\boxed{E = \dfrac{\sigma}{\epsilon_0}}$